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$\mathcal{N} = 1$ M5-brane geometries

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Abstract: We describe how to construct solutions to eleven-dimensional supergravity corresponding to M5-branes wrapped on holomorphic 2-cycles embedded in $\mathbb{C}^3$. These solutions preserve $\mathcal{N} = 1$ supersymmetry in four dimensions. In the near-horizon limit they are expected to be dual to $\mathcal{N} = 1$ large-N gauge theories in four dimensions by Maldacena's duality.

Keywords: M-Theory, Brane Dynamics in Gauge Theories, AdS/CFT Correspondance, Supergravity Models.

Contents

1. Introduction

Maldacena's AdS/CFT conjecture [1, 2] relates gauge theories realised as world-volume field theories on branes to supergravity in the near-horizon geometry produced by those branes. Since it is known how to describe large classes of supersymmetric field theories using brane configurations, it is of interest to find the corresponding near-horizon supergravity solutions. In this paper we investigate the geometry produced by M5-branes wrapped on Riemann surfaces embedded in $\mathbb{C}^3$. Particular choices of Riemann surfaces describe intersecting M5-branes. Finding the supergravity description of fully localised intersecting branes is a topic of current interest [3]–[20]. However, our main motivation is to find supergravity duals of a class of $\mathcal{N} = 1$ gauge theories since the latter share many features with non-supersymmetric QCD. Finding the supergravity solutions, at least in the near horizon limit, should provide a tool to understanding the strong coupling limit of these field theories.

Alternative methods for constructing supergravity duals of field theories, such as the near-horizon limit of D3-branes in a non-trivial type-IIB background or as solutions of five-dimensional gauged supergravity, are described in [34].

Recently progress has been made in understanding the supergravity description of wrapped branes in various contexts. The closely related $\mathcal{N} = 2$ preserving backgrounds describing wrapped M5-branes on 2-cycles in $\mathbb{C}^2$ were found in [21], and the corresponding problem for NS5-branes in type-IIA theory was studied in [19]. M5-branes wrapping 3-cycles of G2 holonomy manifolds were considered in the recent paper [22], Dp-branes wrapping collapsed cycles in orbifold compactifications were described in [23, 24], and branes wrapping Special lagrangian submanifolds were treated in [25]. Many other related examples of warped compactifications or branes wrapped on supersymmetric cycles have been considered in [26]–[32].

Some features of a large class of supersymmetric "compactifications" (on non-compact manifolds) of eleven-dimensional supergravity were discussed in [33]. The supergravity solutions of an M5-brane wrapped on a Riemann surface embedded in $\mathbb{C}^2$ or $\mathbb{C}^3$ are special cases of these compactifications. In [33] a number of constraints on supersymmetry preserving geometries are provided which our explicit solutions satisfy.

The outline of this paper is as follows. We generalise to $\mathcal{N}=1$ the construction in [3, 4, 21], which describes the eleven-dimensional supergravity duals of $\mathcal{N}=2$ field theories as the near-horizon limit of an M5-brane wrapped on a Riemann surface $\Sigma \subset \mathbb{C}^2$. On general grounds it is expected that the supergravity description will be valid in the large-N limit. The number of colours N is related to the genus of Σ but more work needs to be done to establish the precise region of validity of the supergravity duals. As we will review in section 2, the eleven-dimensional supergravity dual of certain $\mathcal{N}=1$ field theories (so-called MQCD theories which are in the same universality class as supersymmetric Yang-Mills theories) is given by the near-horizon limit of an M5-brane wrapped on a Riemann surface $\Sigma \subset \mathbb{C}^3$. Again we postpone the consideration of the precise conditions for the supergravity solution to be a good description of the field theory and concentrate on first providing a method to find explicit solutions to the supergravity equations of motion. We perform the first steps of this task in section 3 where we explain how to construct the supergravity solution by solving the appropriate BPS conditions.

2. M5-brane setup

Brane configurations describing $\mathcal{N}=1$ field theories, which are generalisations of the $\mathcal{N}=2$ configurations of [35, 36, 37], have been constructed in [38]–[41]. The world-volume theories were dubbed "MQCD" to distinguish them from QCD which does not share all the properties of the world-volume theories.

The idea is to begin with a system of NS5-branes and D4-branes in type-IIA string theory. In the simplest case there are two NS5-branes, denoted by $\mathrm{NS5}_1$ and $\mathrm{NS5}_2$. The $\mathrm{NS5}_1$ brane has world-volume directions 012345, while the $\mathrm{NS5}_2$ brane has world-volume directions 012389. The $\mathrm{NS5}_1$ and $\mathrm{NS5}_2$ branes are separated in the 6 direction and we define the $\mathrm{NS5}_1$ brane to be to the left of the $\mathrm{NS5}_2$ brane. One can now consider D4-branes of three distinct types, all with world-volume directions 01236. We have n_L D4-branes with semi-infinite extent in the 6 direction ending on the left of the $\mathrm{NS5}_1$ brane and similarly n_R semi-infinite D4-branes ending on the right of the $\mathrm{NS5}_2$ brane. Finally n D4-branes of finite extent in the 6 direction are suspended between the $\mathrm{NS5}_1$ and $\mathrm{NS5}_2$ branes. This configuration describes an $\mathcal{N}=1$ four-dimensional $\mathrm{SU}(n)$ field theory on the worldvolume of the n finite D4-branes. The semi-infinite branes contribute matter in both the fundamental and anti-fundamental representations of the gauge group. So there are $n_L + n_R$ flavours and the theory is non chiral.

More generally the $\mathrm{NS5}_2$ brane can be be partially rotated (or sheared [42]) from the 45 plane into the 89 plane. This can be seen as a deformation of the $\mathcal{N} = 2$ theory (with the two NS5-branes parallel) by turning on a mass for the adjoint scalar in the $\mathcal{N} = 2$ vector multiplet, breaking the supersymmetry to $\mathcal{N} = 1$. As in the $\mathcal{N} = 2$ configurations, more general setups can be considered by adding more NS5-branes with D4-branes between them, describing an $\mathcal{N} = 1$ field theory with gauge group $\prod_i \mathrm{SU}(n_i)$ and bi-fundamental matter. The NS5-branes can be rotated by different amounts between the 45 and 89 planes, although arbitrary rotations do not preserve supersymmetry (see for instance [43]). These configurations can be lifted to M-theory where they become an M5-brane wrapped on a non-compact Riemann surface Σ embedded in $\mathbb{C}^3$, generalising the $\mathcal{N} = 2$ case of $\Sigma \subset \mathbb{C}^2$.

Further generalisations are possible, introducing D6-branes or orientifold planes which we will not explicitly consider in this paper. These generalisations allow the description of field theories with orthogonal and symplectic gauge groups and more general matter contents including chiral theories. There have been many papers investigating properties of these brane configurations and their relevance to field theory. We simply refer the reader to the general review of brane configurations describing field theories in various dimensions [43].

3. Supersymmetry preserving solutions

In this section we present supersymmetry preserving solutions of eleven-dimensional supergravity relevant for describing the M5-brane set-up described above. Our starting point is a metric, generalising that of [4], with the isometries of the brane configuration:

$$ds^2 = H_1^2 \eta_{\mu\nu} dx^\mu dx^\nu + 2G_{M\overline{N}} dz^M dz^{\overline{N}} + H_2^2 dy^2 \,. \tag{3.1}$$

Here the x^μ are coordinates in four-dimensional Minkowski space and $\eta_{\mu\nu}$ is the flat Minkowski metric. The z^M are holomorphic coordinates:

$$\begin{aligned} z^1 &= v = x^4 + ix^5 \,, \\ z^2 &= w = x^6 + ix^7 \,, \\ z^3 &= s = x^8 + ix^9 \,. \end{aligned} \tag{3.2}$$

Four-dimensional Lorentz symmetry requires that the metric not depend explicitly on the x^μ: thus $H_1, H_2, G_{M\overline{N}}$ depend solely on $z^M, z^{\overline{N}}$ and y. We are interested in holomorphic curves in the six-dimensional space spanned by the z^M, motivating the use of complex coordinates. In addition we have assumed that the metric in this sub-space is hermitian.

The strategy is as follows. We are interested in finding supersymmetric solutions of eleven-dimensional supergravity. The bosonic fields of the theory are the metric and a 3-form potential, while the gravitino is the only fermionic field in the theory. We set the gravitino field to zero and look for supersymmetry preserving solutions by setting the variation of the gravitino under supersymmetry to zero. This fixes the 4-form field strength of the 3-form gauge potential in terms of the metric appearing in (3.1) as in [4].

The gravitino variation is:

$$\delta\Psi_i = D_i\epsilon + \frac{1}{144}\Gamma_i{}^{jklm}F_{jklm}\epsilon - \frac{1}{18}\Gamma^{jkl}F_{ijkl}\epsilon\,. \tag{3.3}$$

The supersymmetry variation parameter, ϵ, for our brane configuration was determined in [46] where the supersymmetry properties of precisely this configuration were investigated using the methods of [45].[1] In that work it was established that the brane configuration is invariant under the variation parameter with the following properties:

$$\begin{aligned}
\epsilon &= \alpha + \beta\,,\\
\beta &= B\alpha^*\,,\\
i\hat\Gamma_0\hat\Gamma_1\hat\Gamma_2\hat\Gamma_3\alpha &= \alpha\,,\\
i\hat\Gamma_0\hat\Gamma_1\hat\Gamma_2\hat\Gamma_3\beta &= \beta\,,\\
\hat\Gamma_{\overline{m}n}\alpha &= \eta_{\overline{m}n}\alpha\,,\\
\hat\Gamma_{\overline{m}n}\beta &= -\eta_{\overline{m}n}\beta\,,
\end{aligned} \tag{3.4}$$

$\eta_{\overline{m}n}$ is the flat euclidean metric, hatted gamma matrices satisfy the flat space Clifford algebra, and B is the charge conjugation matrix:

$$\hat\Gamma^* = B\hat\Gamma B\,. \tag{3.5}$$

To proceed we set (3.3) to zero with the ansatz for the metric (3.1) making use of the properties of ϵ given in (3.4).

Setting the supersymmetry variation in (3.3) to zero results in a set of equations relating the space-time fields to each other. These relations can be reduced to the following set of independent equations:

$$\partial_y \ln H_1 = -\frac{1}{12}\partial_y \ln\det G\,, \tag{3.6}$$

$$\partial_y \ln H_2 = \frac{1}{6}\partial_y \ln\det G\,, \tag{3.7}$$

$$\partial_{\overline{Q}} \ln H_2 = -2\partial_{\overline{Q}}H_1\,, \tag{3.8}$$

[1] See also [44].

$$
\begin{aligned}
F_{M\overline{Q}\overline{L}y} &= \frac{1}{2}\{\partial_{\overline{L}}(H_2 G_{M\overline{Q}}) - \partial_{\overline{Q}}(H_2 G_{M\overline{L}})\}\,, && (3.9)\\
G^{M\overline{P}}G^{N\overline{Q}}F_{MN\overline{P}\overline{Q}} &= -\frac{1}{2}H_2^{-1}\partial_y \ln\det G\,, && (3.10)\\
F_{M\overline{N}\overline{P}\overline{Q}} &= 0\,, && (3.11)\\
F_{y\overline{N}\overline{P}\overline{Q}} &= 0\,, && (3.12)\\
\partial_{\overline{L}}(H_2^2 G^{M\overline{L}}) &= 0\,. && (3.13)
\end{aligned}
$$

In addition there are equations involving derivatives of the variation parameters which, after using the above equations, can be reduced to:

$$
\begin{aligned}
\partial_i\alpha + \frac{1}{4}\partial_i \ln H_2 &= 0\,,\\
\partial_\mu \alpha &= 0\,, && (3.14)
\end{aligned}
$$

where i can be $y, M, \overline{M}$ and μ is an index in the 0123 directions. We have only listed independent relations, the remaining follow from requiring that the four-form F is real, and that α is related to β through (3.4).

Taking into account the observations of [11, 26],

$$
H^2|f(z)|^2 \equiv H_2^6|f(z)|^2 = |\det G| \tag{3.15}
$$

with f a holomorphic function of z^M. The arbitrariness of the function f allows for the freedom to make a holomorphic change of variables in z^M [26]. In the following we will use coordinates where $f(z) = 1$.

In addition to the supersymmetry conditions there are constraints on the fields arising from the Bianchi identity and equation of motion of the four-form field strength. Since we are considering a geometry produced by M5-branes, which couple magnetically to the three-form potential, the roles of the Bianchi identity and equation of motion are reversed. Therefore we require that $d * F = 0$ trivially. This determines $F_{MN\overline{P}\overline{Q}}$ in terms of the metric:

$$
F_{MN\overline{P}\overline{Q}} = \frac{1}{2}\partial_y[H_2^{-1}(G_{N\overline{P}}G_{M\overline{Q}} - G_{N\overline{Q}}G_{M\overline{P}})]\,. \tag{3.16}
$$

Finally, we write down the source equation for F:

$$
\begin{aligned}
J_{ML\overline{K}\overline{N}y} = (dF)_{ML\overline{K}\overline{N}y} = {} & [\partial_M\partial_{\overline{N}}(H_2 G_{L\overline{K}}) - \partial_M\partial_{\overline{K}}(H_2 G_{L\overline{N}}) -\\
& - \partial_L\partial_{\overline{N}}(H_2 G_{M\overline{K}}) + \partial_L\partial_{\overline{K}}(H_2 G_{M\overline{N}})]\\
& - \frac{1}{2}\partial_y^2\{H_2^{-1}(G_{M\overline{K}}G_{L\overline{N}} - G_{L\overline{K}}G_{M\overline{N}})\}\,, && (3.17)
\end{aligned}
$$

where J is the source 5-form specifying the Riemann surface on which the M5-brane is wrapped. The other components of dF vanish when the constraint on the metric (3.13) is taken into account.

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The solution is expressed in a more elegant form in terms of the rescaled metric:

$$g_{M\overline{N}} = H^{-1/6} G_{M\overline{N}} , \tag{3.18}$$

and its associated hermitian 2-form:

$$\omega = i g_{M\overline{N}} dz^M \wedge dz^{\overline{N}} . \tag{3.19}$$

In terms of the rescaled metric:

$$\begin{aligned} ds^2 &= H^{-1/3}\eta_{\mu\nu}dx^\mu dx^\nu + 2H^{1/6} g_{M\overline{N}} dz^M dz^{\overline{N}} + H^{2/3} dy^2 , \\ \det g &= H , \\ F &= \partial_y(\omega\wedge\omega) - i\partial(H^{1/2}\omega)\wedge dy + i\overline{\partial}(H^{1/2}\omega)\wedge dy , \\ \overline{\partial}(\omega\wedge\omega) &= 0 . \end{aligned} \tag{3.20}$$

In the above equations ∂ denotes the (1,0) exterior derivative $\partial = dz^M \partial_M$ in the subspace spanned by the z^Ms. Notice that the constraint on the metric (3.13) is transformed into the property that the 4-form ω^2 is closed. Finally, the equation of motion for F is written simply as:

$$dF = \partial_y^2(\omega\wedge\omega)\wedge dy - 2i\overline{\partial}\partial(H^{1/2}\omega)\wedge dy = J , \tag{3.21}$$

where J again denotes the source 5-form.

As a consistency check one can easily see that the $\mathcal{N}=2$ [4] solution satisfies the above constraints. Another check is provided by the recent work of Becker and Becker [33] who found constraints on supersymmetric M-theory backgrounds with four-dimensional Lorentz invariance and four-form flux. Our solution satisfies their equations, which is a non-trivial test.

4. Conclusions and discussion

In this paper we have found supersymmetry preserving solutions of eleven-dimensional supergravity involving M5-branes wrapping 2-cycles in $\mathbb{C}^3$. The supergravity fields are expressed in terms of an auxiliary hermitian metric and its associated two-form. The two-form satisfies a constraint as well as a source equation.

The main motivation for studying wrapped M5-brane configurations comes from Maldacena's conjecture relating supergravity in near-horizon black hole geometries to Quantum Field Theories. $\mathcal{N}=1$ gauge theories are the most realistic supersymmetric theories in that they display many of the features evident in ordinary QCD such as confinement and chiral symmetry breaking. Recent progress in identifying supergravity duals for $\mathcal{N}=1$ theories in IIB [32, 47, 48, 49] and M-theory [50, 51] has resulted in new ways of understanding field theory phenomena.

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The source term in eq. (3.21) specifies the cycle on which the M5-brane is wrapped. This, in turn, determines the dual gauge theory. To complete the program of finding geometries dual to interesting gauge theories, we must solve the source equation for appropriate two-cycles. In this paper we do not attempt to solve the source equation, but we are heartened by the success of our attempts at solving such equations in the $\mathcal{N}=2$ context [21]. In that work the key observation which made finding the solution possible, was that locally one can treat the M5-brane as being flat. Then the local geometry in terms of appropriate variables is that of an ordinary flat M5-brane. We believe that these observations will play an important role in solving the $\mathcal{N}=1$ case as well. One of the crucial simplifications in the $\mathcal{N}=2$ case was to directly find the solution in the near-horizon limit, rather than finding the full asymptotically flat solution first. We expect this will also simplify the procedure in the $\mathcal{N}=1$ case and of course the near-horizon solution is all we are interested in finding for the purposes of find the supergravity duals. We hope to return soon to the question of finding explicit solutions dual to interesting field theories.

Another motivation for studying wrapped M5-brane configurations comes from the Randall-Sundrum [52] approach to solving the hierarchy problem. There, warped geometries of the kind we display play a crucial role. Moreover, recent results [31] indicate that string theory realizations of the Randall-Sundrum scenario miss important physics unless the full 10- or eleven-dimensional geometry is taken into account. This problem arises due to the appearance of singularities in geometries of lower dimensional truncated supergravity, even in situations in which the ten- or eleven-dimensional geometry is singularity free. Thus finding interesting $\mathcal{N}=1$ geometries relevant for the Randall-Sundrum scenario may be of phenomenological interest.

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